Pure Core 3 Past Paper Questions Pack B

Taken from MAP2

June 2001

1 Find $\frac{dy}{dx}$ for each of the following cases

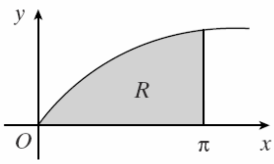
(a) $y = e^{2x} \sin 3x$, *(3 marks)*

(b) $y = (2x^2 + 1)^5$. *(2 marks)*

7 (a) Sketch, on the same diagram, the graphs of $y = \ln x$ and $y = \frac{3}{x}$ for $x > 0$. *(2 marks)*

(b) (i) Show that the equation $\ln x - \frac{3}{x} = 0$ has a root between $x = 2$ and $x = 3$. *(2 marks)*

8 The graph below shows the region R enclosed by the curve $y = x + \sin x$, the x-axis and the line $x = \pi$.



(a) Find the exact value of the area of the region R. *(4 marks)*

(b) Show that

(i) $\int_0^{\pi} x \sin x \, dx = \pi$, *(4 marks)*

(ii) $\int_0^{\pi} \sin^2 x \, dx = \frac{\pi}{2}$. *(4 marks)*

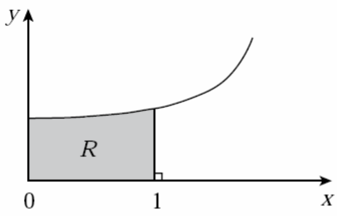
(c) A solid metallic casting is made by rotating the region R through 2π radians about the x-axis. Find the volume of the solid formed. *(3 marks)*

January 2002

3 Find the equation of the tangent to the curve $y = \frac{2 + x}{\cos x}$ at the point on the curve where $x = 0$. *(6 marks)*

7 (a) Express $\frac{1}{2-x} + \frac{1}{2+x}$ in the form $\frac{A}{4-x^2}$ where A is a constant. *(2 marks)*

Part of the graph of $y = \frac{1}{\sqrt{4-x^2}}$ is shown below.



(b) Using the result of part (a), show that the exact volume of the solid formed when the shaded region R is rotated through 2π radians about the x-axis is

$$\frac{\pi \ln 3}{4}.$$ *(6 marks)*

(c) (i) By using the substitution $x = 2\sin\theta$ show that

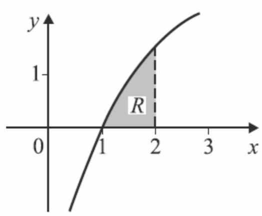
$$\int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\left(\frac{x}{2}\right) + C,$$

where C is a constant. *(4 marks)*

(ii) Hence find the area of the shaded region R. *(2 marks)*

June 2002

4 The graph below shows the region R enclosed by the curve $y = x - \frac{1}{x}$, the x-axis and the line $x = 2$.



Find the exact volume of the solid formed when the region R is rotated through 2π radians about the x-axis. *(6 marks)*

5 (a) Find $\frac{dy}{dx}$ when:

(i) $y = x \tan 3x$; *(3 marks)*

(ii) $y = \frac{\sin x}{x}$. *(3 marks)*

(b) Show that $\int_0^{\frac{\pi}{8}} x \sin 2x \, dx = \frac{4 - \pi}{16\sqrt{2}}$. *(6 marks)*

January 2003

2 (a) Show that

$$\int_0^6 \frac{1}{2+u} \, du = \ln n,$$

where n is an integer to be found. *(3 marks)*

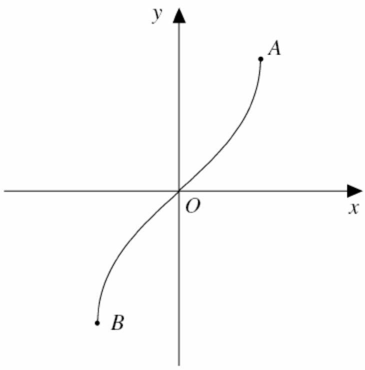
(b) Use the substitution $x = u^2$ to show that

$$\int_0^{36} \frac{1}{\sqrt{x}(2+\sqrt{x})} \, dx = \ln m,$$

where m is an integer to be found. *(5 marks)*

5 (a) The diagram shows the graph of

$$y = \sin^{-1} x.$$



Write down the coordinates of the end-points A and B. *(2 marks)*

(b) Use the mid-ordinate rule, with five strips of equal width, to estimate the value of

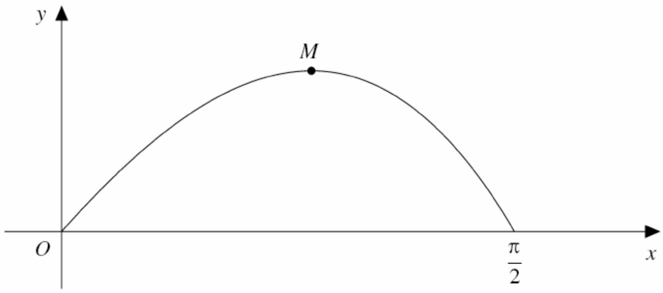
$$\int_0^1 \sin^{-1} x \, dx.$$

Give your answer to three decimal places. *(5 marks)*

7 The diagram shows a sketch of the curve

$$y = x\cos x, \quad 0 \leqslant x \leqslant \frac{\pi}{2}.$$

The maximum point is M.



(a) (i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Show that the x-coordinate of M satisfies the equation

$$x = \tan^{-1}\frac{1}{x}.$$ *(3 marks)*

(iii) It is estimated that the relevant root of the above equation is approximately 0.9.

Use the iterative formula

$$x_{n+1} = \tan^{-1}\frac{1}{x_n},$$

starting with $x_1 = 0.9$, to obtain the root correct to two decimal places. *(3 marks)*

(b) Find the area of the region bounded by the curve and the x-axis. *(6 marks)*

June 2003

1 Use integration by parts to find

$$\int_0^{\frac{1}{2}} x\,e^{2x}\,dx.$$ *(5 marks)*

5 A curve has the equation

$$y = \frac{2x}{\sin x}, \qquad 0 < x < \pi.$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) The point P on the curve has coordinates $\left(\frac{\pi}{2}, \pi\right)$.

(i) Show that the equation of the tangent to the curve at P is $y = 2x$. *(3 marks)*

(ii) Find the equation of the normal to the curve at P, giving your answer in the form $y = mx + c$. *(3 marks)*

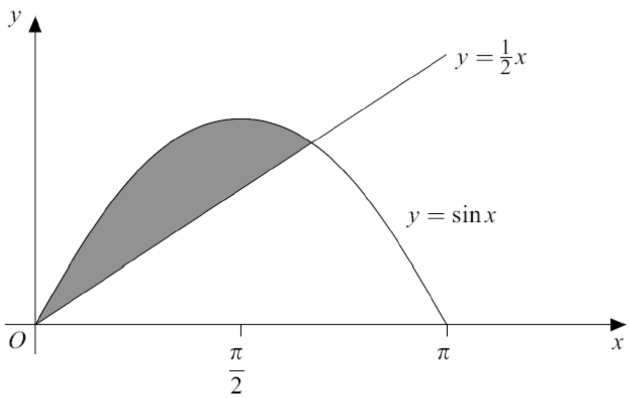
January 2004

4 (a) By using the chain rule, or otherwise, find $\frac{dy}{dx}$ when $y = \ln(x^2 + 9)$. *(3 marks)*

(b) Hence show that $\int_0^3 \frac{x}{x^2+9} dx = \frac{1}{2}\ln 2$. *(3 marks)*

(c) Show that $\int_0^3 \frac{x+1}{x^2+9} dx = \frac{1}{2}\ln 2 + \frac{\pi}{12}$. *(4 marks)*

6 The diagram below shows the graphs of $y = \sin x$ and $y = \frac{1}{2}x$, for $0 \leqslant x \leqslant \pi$.



(a) Show that the equation $\sin x - \frac{1}{2}x = 0$ has a root in the interval $1 \leqslant x \leqslant 2$, where x is measured in radians. *(2 marks)*

(b) (i) Given that $f(x) = \sin x - \frac{1}{2}x$, find $f'(x)$. *(1 mark)*

(c) (i) Show that $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$. *(2 marks)*

(ii) Hence find $\int_0^{1.9} \sin^2 x \, dx$. *(1 mark)*

(d) The shaded region enclosed by the graph of $y = \sin x$ and the line $y = \frac{1}{2}x$ is rotated through one revolution about the x-axis to form a solid.

Calculate an approximation for the volume of this solid, giving your answer to two significant figures. *(5 marks)*

June 2004

3 (a) Use integration by parts to evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$. *(5 marks)*

(b) (i) Use the substitution $t = x^2 + 4$ to show that $\int \frac{2x\,dx}{\sqrt{x^2+4}} = \int \frac{1}{\sqrt{t}} dt$. *(2 marks)*

(ii) Show that $\int_0^2 \frac{2x\,dx}{\sqrt{x^2+4}} = 4(\sqrt{2} - 1)$. *(4 marks)*

4 (a) (i) Find $\frac{dy}{dx}$ when $y = e^x \sin 2x$. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve $y = e^x \sin 2x$ at the origin. *(2 marks)*

(b) Show that the equation of the normal to the curve $y = e^x \sin 2x$ at the point where $x = \pi$ is

$$2e^{\pi} y + x = \pi.$$ *(4 marks)*

5 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

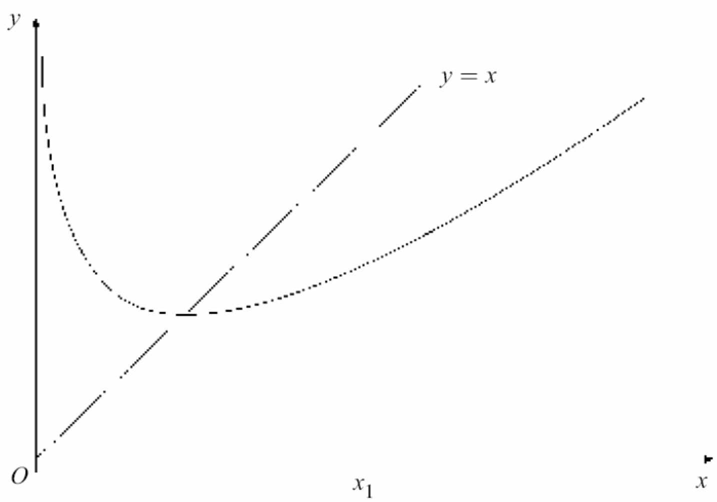
(a) Show, without using a calculator, that the equation $x^3 - 15 = 0$ has a root in the interval $2 \leqslant x \leqslant 3$. *(2 marks)*

(b) (i) Show that the equation $x = \frac{2x}{3} + \frac{5}{x^2}$ can be rearranged to give the equation

$$x^3 - 15 = 0.$$ *(2 marks)*

(ii) Use the iterative formula $x_{n+1} = \frac{2x_n}{3} + \frac{5}{x_n^2}$, starting with $x_1 = 3$, to find the values of x_2, x_3 and x_4, giving your answers to six decimal places. *(4 marks)*

(iii) The graphs of $y = \frac{2x}{3} + \frac{5}{x^2}$ and $y = x$ are sketched below.



On **Figure 1**, draw a staircase diagram to illustrate the convergence of the sequence $x_1, x_2, x_3, \ldots$ *(2 marks)*

(iv) Write down the **exact** value to which this sequence converges. *(1 mark)*